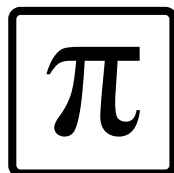


Milestone Maths
by
Kathy Gonzalez

Teacher Book
Level B



Milestone

For Joshua

My youngest son and the inspiration for this series.

Milestone Maths Teacher Book B

First Edition (2024)

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Introduction

Welcome to Milestone Maths. This course has been developed with Australian home schoolers particularly in mind but will also be useful in the classroom or for after school study with or without a tutor. The first year is necessarily teacher led, but even here the familiar layout and simple presentation will allow children to quickly develop independence for many parts of each lesson.

This course is gradual, systematic and thorough. Mathematics is a sequential subject where one concept is built upon another and thorough mastery of each step is essential for true understanding of the whole. This is reflected in Milestone Maths by presenting new topics sequentially and in a manner that builds from the known to the unknown. Review is built into the program and the needs of students with different abilities are catered for by pacing guidelines and supplementary practice activities.

The scope and sequence roughly follows the Australian Curriculum mathematics syllabus. However, for tutors and homeschool parents, I would strongly recommend that the child be placed at the level where their understanding begins. In most cases, a child new to the program should begin at the first book, working quickly through the milestones until they find the point where they are learning new material.

This teacher manual has been kept as brief as possible by design: as a busy homeschool mum, I understand that you don't have hours to devote to lesson preparation. I trust you will find the information contained valuable.

Acknowledgments

Sincere thanks go to John Garrad and the LEM team for their invaluable assistance towards making this project a reality; this program far exceeds my initial expectations because of their advice and efforts. Thanks are also due to all my students, past and present; each one has made me a better teacher. I am also grateful to the many friends who encouraged me to stop procrastinating and bring my vision to fruition. And finally many thanks to my family for their efforts, support and advice on various aspects of this program as well as bearing patiently with the change of routine that such a project inevitably involves.

How to Master Mathematics

I hear and I forget. I see and I remember. I do and I understand.

Chinese Proverb

Mathematics is one of the very few, if not only, truly universal languages. Thus teaching and practising mathematics is primarily about learning a new language and applying a set of skills. It is very much like learning to play a musical instrument or a sport. Few would argue that a person can reach their full potential in music or sport simply by reading or hearing about the chosen instrument or game. The same is true of mathematics.

Since mathematics is in fact a language, let's consider how your child first learned to speak their "mother tongue." From the first dawning of intelligence your youngster tirelessly practised various gurgles and babbles preparing the mind and speech organs for the task of ever increasingly sophisticated communication. What began as "wah, wah, waaaahhh," developed into "Milk!" which hopefully developed into, "Mum, may I please have some milk?" and may sadly one day retrograde into, "Where's the milk?!?" or perhaps a lone grunt accompanied by an indistinct motion towards the milk bottle.

The important point in this progression is that every step, but most assuredly the early stages, is marked by practice, practice, practice. The reason your toddler drives you round the bend repeating odd, disconnected words and phrases is that he is practising his speech skills. Your young child drives you no less crazy asking "Why...?" or "What happens if...?" questions not so much to hear the answers as to practise conversational skills.

Likewise, if you want to become proficient in, and truly understand mathematics, you must practise just as much as you did when acquiring your first language skills. In fact, probably more because, unlike your mother tongue, mathematics is not a language in which you are constantly immersed. Virtually everyone I know who enjoys mathematics had a parent, grandparent or teacher who encouraged (or coerced) their charge to practise the basic skills of arithmetic to the point of mastery. Mastery in this context means over learning to the point that the skill becomes as automatic as speaking your name.

Once this simple step — of over learning of the basic facts and skills of arithmetic — is taken, an unshakeable foundation is laid. As a bonus, a fascination with numbers is often born the fruit of which may know no bound and will lead to a life of learning and discovery. Since mathematics is the foundational language of all true science, there is truly no limit to the wonders awaiting discovery by the enquiring young mind.

Milestone maths is built around the idea of practice, practice, practice until true mastery is achieved. The student book lessons and drills work together to build fluency in the foundational skills essential for true mathematical mastery.

With the above discussion in mind, please do not consider the reviews and drills an unnecessary optional extra. The master musician or medal winning athlete only achieved success through hours of practice of the apparently minor skills: the musician played many thousands of scales and the athlete completed many thousands of laps around a track or down the lane of a pool. Play the role of a coach and encourage your child to make the drills a pleasant, or at the least painless, experience. Use a timer if that will motivate your child but put it away if it makes your child nervous or stressed. Make a game of it – challenge the child to a race: you can complete the same activity at the same time but with the handicap of having to write the exercises out on blank paper as you answer them. A little care and attention to building right habits of thinking and working from the beginning will pay huge dividends and will ensure your child's future success in maths.

How to Use This Course

This course is intended to be comprehensive and “stand alone.” It is also designed to be “open-and-go” with the busy homeschool parent in mind. At this level level, it is assumed that the child will not be reading independently, so the parent will necessarily need to be fairly heavily involved in delivering lessons. However, lessons are designed to be fairly short (15-20 mins) and much of the written work is designed so that it can, and should, be completed independently by the child.

Where to Start and How to Progress

To begin at level B a child should be able to count to 10 and recognise the written numbers 1-10. They should also be fairly confident at counting out a given number of objects from a larger collection, although this will be reviewed and reinforced. They do not need to be able to write the numbers.

While it is impossible to make one fixed series of lessons to meet the needs of all students, this series is designed to be flexible enough fulfill the needs of the majority of students. With this in mind, please note that you do not need to finish the entire “B” series in exactly one school year. If your child is finding the concepts easy, you may consider doing two lessons in one day and conversely, if the subject matter is more challenging, break the lesson up across two or more days. Also, lessons/activities marked with a graduation cap icon are considered advanced material to challenge the more motivated/talented students. Advanced material will usually become ‘mainstream’ at some point later in the course.



Materials Required

To make this course accessible and simple to use, the required materials have been deliberately kept to a minimum. The essential materials required to complete the lessons in this level are:

Student Workbooks. The student workbooks are the backbone of this curriculum. They give the average student sufficient practice to understand the concepts being taught. Student workbooks are consumable so one copy must be purchased for each student.

The Teacher Book. This teacher’s guide will give you instructions on how to teach the lessons as well as guidance on how to adapt the curriculum to the particular needs of your child.

A set of Sumstix. See page 6 for details.

Number Bond Flashcards. The Milestone Maths deck of number bond flashcards covers both addition/subtraction and multiplication/division and is numbered to correspond to the order of study in the course. They are available for purchase from LEM.

Optional Resources

Number and Game Cards. The Milestone Maths Number and Game Cards contain numbers and symbols that are used to give “hands on” practice with various number skills. In this level, only the numbers from 0-10 will be used so the 0-10 Game Card option below may be used as an alternative, however you will have to make your own symbol cards for '+', '-', and '='.

If you already have the following two resources, they may be used as an alternative to the number and game cards.

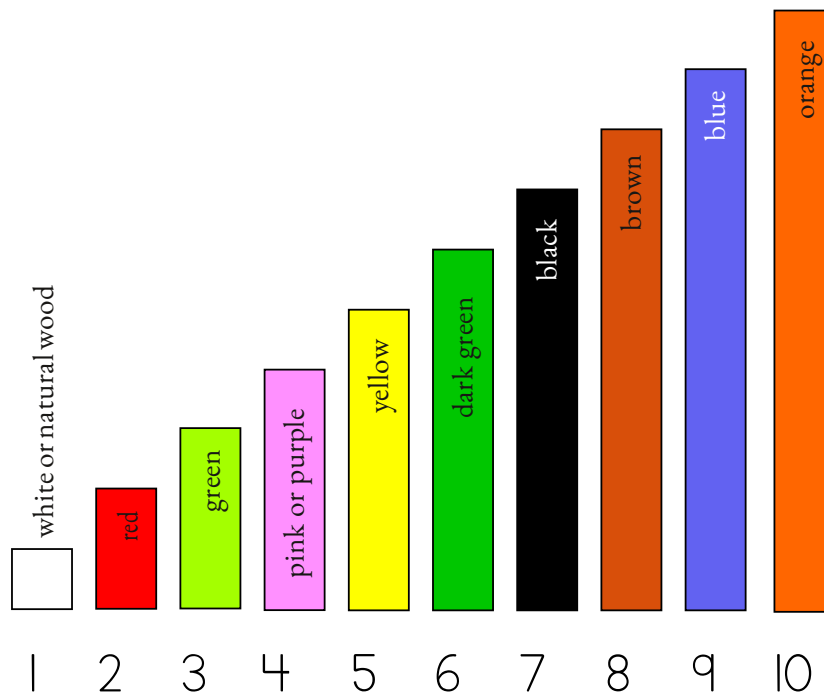
Number Cards. A deck of number flashcards for the numbers 0-100, preferably single sided. These will be used more heavily in level C.

0-10 Game Cards. These may be a deck of standard playing cards with the queen and king cards removed (the jacks count as zero). They may also be cards from the various number based card games commercially available.

Sumstix™

What are Sumstix™?

Sumstix™ are compatible with Cuisenaire rods which were invented by a Belgian teacher in the 1930s. The name "Sumstix" is read the same whether it refers to singular or plural, that is 1 sumstix or many sumstix. Sumstix consist of "sticks" cut from a 1cm square of timber in lengths ranging from 1cm to 10cm. Each length is a distinct colour and may be used to represent a number as follows:



Why Use Sumstix™?

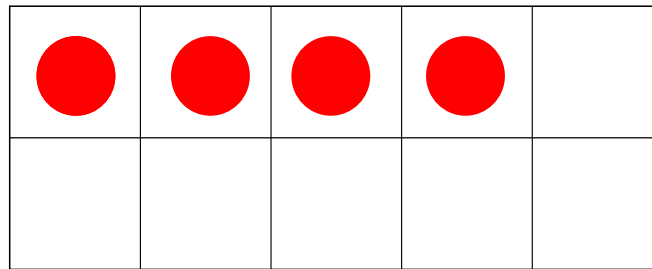
Sumstix offer children a concrete way to represent numbers. Numbers are an abstract representation of actual quantities, so, for example, we use the number 5 to tell how many fingers we have on one hand. Our modern number system and the operations of arithmetic allow us to conveniently work with such quantities to quickly determine the number of objects in more than one group of objects. Continuing our example, one person has $5 + 5 = 10$ fingers on both hands and 4 people have $10 + 10 + 10 + 10 = 4 \times 10 = 40$ fingers all together.

While it is necessary to teach children first to count by assigning numbers to collections of discrete objects, it is counterproductive to continue reinforcing this association once the operations of arithmetic are introduced. This is because if you show a child that $5 + 5$ can be found by counting the total number of objects in two collections of five objects, they will latch onto this idea and use counting (or "counting on from 5") as their sole "addition strategy" habitually. We shall refer to this process of counting to solve addition problems as "unit counting."

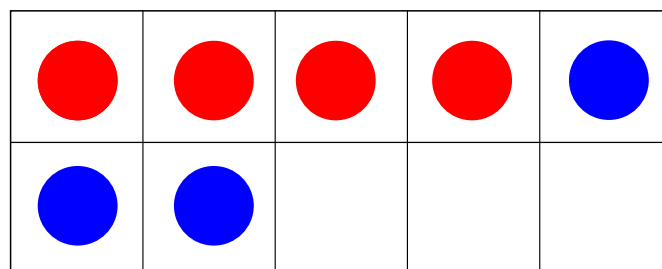
However, taking the effort to actually memorise the addition and multiplication tables is vital for a child to succeed in understanding and learning any maths beyond the very basics of counting and simple addition. Further, contrary to popular belief, this task is really no different to, and therefore no more difficult than, memorising the sequence of numbers in the first place (that is, learning to count). This is not obvious nor relevant to the young beginner and so it is up to the educator to make the process of learning to count in new ways (that is, via memorised addition or multiplication facts) intuitive and more convenient than unit counting.

Along this line, some researchers and educators have observed that a person's skill at subitizing, or the ability to tell the number of a small collection of objects at sight (that is, without counting) appears to be predictive of their ability to learn and perform arithmetic. While the skill may be taught and practised, it does have psychological and physical limitations: most adults are only able to subitize 3 or 4 objects, for example. However, to be truly useful in the context of arithmetic calculations, subitizing needs to occur at least to 5, and preferably to 10.

It is possible to increase the number of objects that can be subitized by presenting them in a fixed pattern such as on dice or dominoes. In mathematics education a ten frame is often used to present numbers in a way that is easy to subitize. A child need only learn to subitize to 2 or 3 in order to easily tell the number of objects in the frame.



Most children develop the ability to subitize to 2 naturally at a very young age so this is a fairly natural progression. However, the limitation of a ten frame becomes obvious when we use it to present an addition problem like this:



Most children will resort to unit counting to find the number of blue dots in the above illustration and thus defeat the purpose of using the ten frame. This is where Sumstix come in. Children acquire the ability to compare lengths much earlier than the ability to compare numbers of objects by subitizing. There is also no physical limitation to this ability as there is with subitizing.

Numbers are actually an abstract way of "chunking" units. The number 3 really means $1 + 1 + 1$ but it is much more convenient and efficient to represent the concept of three with the digit "3" than by some representation of three objects (say, something like |||). Sumstix give the child a concrete way to represent this "chunking." A light green rod is exactly three times the length of a white rod so it may accurately represent the concept of 3 as three units (eg $1 + 1 + 1$ or 3×1).

Further, because of their simple geometry, two or more Sumstix placed end-to-end make an ideal representation of an addition or multiplication expression. Most children quickly learn to estimate length using the rods and this leads to a very intuitive approach to solving basic arithmetic problems. Many children will also begin to visualise the rods after a time of working with them which leads to natural and easy acquisition of the basic number facts.

The visual and concrete nature of Sumstix also makes many other abstract mathematical concepts much easier for the young child to grasp. One such concept is the duality between addition and subtraction. Not only is this idea essential for success in higher level mathematics, it also simplifies the memorisation of the related math facts. While this concept is very abstract and difficult for most children to understand when presented using traditional tools, when it is presented using Sumstix it becomes intuitive and simple for the young child to understand.

In summary, SumstixTM make the learning of mathematics fun and easy for children of all ages.

How to Teach the Lessons

As a busy home school mum, I understand that you don't have the time to read through lengthy lesson plans or to spend hours preparing for every lesson. Hence, the lesson plans have been kept as short and concise as possible and most of the teaching is done directly to the student in the workbook through pictures and text (which you will read at this level). The only preparation required for most lessons is to gather the book and Sumstix.

Basic Structure of Lessons

Each lesson consists of four main parts:

1. **Introduction:** In the first milestone this should consist of a few activities from the Reinforcement and Warm up Activity suggestions. In subsequent milestones, this should include at least one warm up activity followed by a brief review of the previous lesson and could include the child performing a calculation or solving a problem that you specify.

2. **Main lesson:** This is the student book lesson and will generally only require you to read the instructions to the student, complete any practical activities suggested in plan with the student (or lead them to complete the activity on their own) and then allow them to complete the written exercises independently. At this age you will have to read and explain one section at a time.

3. **Independent practice and drill:** In the interest of developing independence as soon as possible, every regular lesson concludes with a Review and Practice section consisting of revision activities. From Milestone 3 your child should be able to complete the majority of the Review and Practice activities independently with little or no assistance from you. Written instructions are kept to a bare minimum and, where possible are replaced by icons which will quickly become familiar to the student.

The Review and Practice section also incorporates the daily drill necessary for true mathematical mastery. "Timed" drills of the basic facts begin in Student Book B2 and are marked with an hourglass icon. Some 'drill like' activities appear in the Review and Practice section from lesson 33 in Student Book B1 which you may treat as timed drills if you wish your child to get used to the idea early.

4. **Checking and feedback:** Once your child has completed the independent bookwork please check the answers and allow them to correct any errors immediately. It is best if you do not supply the correct answer, or even explain exactly where the error is but rather simply point out that there's "something a little bit wrong" with the answer and have the student find it for themselves. This type of immediate self-correction is a powerful learning tool. Ideally, you should check each activity as the child completes it (but avoid "hovering" over the child and correcting each problem as they complete it as this will delay the development of independence). If the child cannot work out where they went wrong, this is a clear sign that they do not understand the concept and that you should re-teach it. Use your discretion to decide whether to give a mini-lesson immediately or whether to spend the next day's lesson entirely on reteaching and practising that concept. Avoid leaving the marking until much later in the day or week as the child will have "moved on" in their mind and the correction procedure will not have the same powerful learning effect.

Importance of Practice and Drills

The importance of the drills cannot be overstated. In my tutoring practice I have seen several students who began to struggle in mathematics from year 5 transformed into "A" students in high school after only one to two years of intensive drilling on the times tables. If the habit of daily practice on the basic facts of numbers is established early, a plethora of problems and struggles will be avoided later on.

If you or your child prefer a more interactive and social approach to achieve the same goal, consider using flashcard drills. The unique Milestone Maths number bond flashcard system is ideal for this purpose because it streamlines the process of learning all the facts for the four basic operations (addition, subtraction, multiplication, and division). Instead of dealing with the full four 10x10 tables, it condenses them into only 110 easy-to-learn number bonds.

Most children will benefit from doing both the written drills and flashcard drills. I recommend using the flashcards as a warm up activity. This will leave some 'space' between the flashcard and written drills. This type of "spaced repetition" is an excellent way to efficiently learn a new skill or to memorise information.

If you school five days per week, studying the number bond flashcards is an ideal Friday activity. Detailed instructions on how to effectively use the number bond flashcards appear in the box with the flashcards.

What are Milestones and Checkpoints?

A Milestone corresponds to a chapter or unit of work. In the student book, the start of each Milestone is marked with a picture of a milestone and the Milestone title. Checkpoints are end of chapter reviews and are essentially lessons dedicated to review and practice of the key concepts and skills introduced in the Milestone. If your child has particular difficulty completing a checkpoint, it is recommended that you spend some time reviewing the concepts taught in the Milestone before moving on. If they have difficulty with only one or two activities, review the concept immediately and make a note to practise those skills often during the introductory phase of subsequent lessons until the skill is mastered.



Instructions for Checkpoints

On lesson days when a checkpoint is reached, introduction/review activities are optional. The child should be able to complete each activity independently, or with very little help, after you have read the instructions. At this level the child may use Sumstix as much as they need to while completing checkpoints, lessons and drills, although they should be encouraged to transition to completing the drills without assistance.

Adaptations for Children with Special Needs

If your child has special learning needs, there are a number of adaptations possible.

For older yet illiterate students: you should read all instructions to the student just as you would to a young child that is still learning to read. To address the literacy issue I would recommend investigating LEM phonics.

For children who have difficulty writing: you may act as scribe and have the child tell you what to write. When numbers or equations are required, have the child "build" the answers using the number and game flashcards. Also, use the number bond flashcards for drills instead of the written drills until writing is easier. Unless the child has a physical handicap that makes writing difficult or impossible, I would suggest that you gently encourage them to do more and more writing on their own every day. Begin by taking turns with the pencil - you write one number then the child writes one, etc and slowly increase the amount of writing that your child does until they achieve independence.

For children who need a slower pace: some lessons could consist entirely of warm up/review activities or the student book activities could be assigned over two or more days.

Extra writing practice can be done on a reusable drawing board (eg whiteboard, LCD tablet, etc.), on scrap paper or in a separate exercise book.

For children who need a faster pace: If your child is finding the lessons very easy and is learning the concepts quickly, you may consider doing two lessons a day and completing the Review and Practice section of only one of the lessons. Special care needs to be taken that the child is mastering the drills at this pace as well. Over learning is always a good thing however, a particularly bright child will need to be challenged to maintain motivation.

Lesson Plans and Teaching Tips

I understand that a busy homeschool parent does not have the time for lengthy lesson planning so the plans on the following pages have been kept to the bare minimum. However, you should consult this book before your child commences each workbook and each Milestone because important teaching tips and enrichment activities are given. In most cases after the first Milestone, it will take less than two minutes to read the introduction and teaching tips for each Milestone. Investing this time will simplify the teaching process for yourself, improve your child's learning experience and will greatly increase the enjoyment and satisfaction that both of you derive from the lessons.

Student Book B1

Student Book B1 lays the essential foundation for all future learning in mathematics. The following concepts are introduced:

- Writing the numbers from 0-10.
- Counting backwards from 10 to 0.
- The number line.
- Counting on from a number.
- The idea that a number (or group) which we call a total can be broken into two smaller smaller numbers (groups) which we call the parts.
- How we use number bonds to represent addition facts.
- How we use Sumstix to represent the numbers.
- How we can use Sumstix to solve simple 2 digit addition problems with a total of 10 or less.
- How to use Sumstix to solve simple addition equations where one of the addends is unknown.

Milestones 1, 2, and 3 collectively form a single comprehensive unit of work. However, instead of presenting it as a single Milestone, it has been divided into three separate Milestones. This deliberate division allows your child additional time to grasp the information thoroughly and become familiar with checkpoints from the outset of this course.

A note on number reversals

In children of this age group, it is quite common to reverse certain numbers while learning to write them. Specifically, they tend to confuse the direction of digits like 2, 3, and 5. This phenomenon occurs because our writing system imposes a constraint that contradicts the child's intuitive understanding of the visual world.

From a very young age, their brains learn to recognise their mother's face regardless of its orientation. Through numerous observations, they develop the idea that all objects exhibit the same pattern. For instance, a triangle remains a triangle whether it appears pointing up or down.

Numbers (and letters) contradict this pattern by imposing the constraint of fixed orientation. For many children this is a concept that takes a long time to learn. It is also very common for a child to write their numbers perfectly one day and then reverse random digits the next. Please do not become frustrated or scold your child for reversing their numbers. Children of this age group are usually trying their very best to please you so scolding them for an unconscious error will do much more harm than good to both the child and your relationship with them. Simply point out the error, erase the number and gently help them (guiding their hand if necessary) to write the number the correct way. With consistent and patient effort number reversals will eventually disappear.

The best way to minimise number reversals is to spend much time practising number formation and stressing the correct formation of each of the digits. For example, insist that zeros be written in a counter clock-wise direction and that fives begin with a down stroke. Attention to detail from the outset will smooth the entire journey.